Discrete Mathematics Seminar

Time: Friday, 8 March 2013, 1:00 – 2:00 PM
Location: 238 Derrick Hall
Title: Two Combinatorial Problems, Only One Solution: A Tale in Two Parts
Speaker: Dr. Jeremy Alm, Department of Mathematics, Illinois College

Part I: Problem Solved!

One of the central results of Euclidean Ramsey Theory is known as Gallai's Theorem:

> Let $\mathcal{S}$ be any finite subset of $\mathbb{R}^n$. Then any finite coloring of $\mathbb{R}^n$ contains a monochromatic subset homothetic to $\mathcal{S}$.

It seems natural to think that *surely* one must be able to find more than one homothetic copy of a given set $\mathcal{S}$. We prove that, in fact, one can find $2^{\aleph_0}$ pairwise-disjoint copies.

Part II: An Unsolved Mystery

Here is the "cyclic basis" problem, a recent topic at this seminar:

> Fix $k \in \mathbb{Z}^+$. Find the largest N such that there exists $A \subseteq \mathbb{Z}_N$, $|A| \leq k$, so that $A \cup (A + A) = \mathbb{Z}_N$.

The following extension of the problem might be called the Cyclic Multi-Basis Problem:

> Fix $m \in \mathbb{Z}^+$. Find the largest N such that there exist pairwise-disjoint subsets $A_1, \ldots, A_m \subseteq \mathbb{Z}_N$ such that each subset A_i is a cyclic basis.

If the subsets A_i have certain strong properties, then the set $\{\{0\}, A_1, \ldots, A_m\}$ forms a set of atoms (minimal non-empty elements) of a so-called *Ramsey Algebra*.

The properties are as follows:

- The A_i's are all *symmetric*
- The A_i's partition $\mathbb{Z}_n \backslash \{0\}$
- $A_i + A_i = \mathbb{Z}_N \backslash A_i$ (so $A \cup (A + A) = \mathbb{Z}_N$ but A and $A + A$ are disjoint)
- For $i \neq j$, $A_i + A_j = \mathbb{Z}_N \backslash \{0\}$.

The question of the existence of (cyclic) Ramsey Algebras is open except for the cases $m = 2$ and $m = 3$.