TEXAS STATE UNIVERSITY

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Discrete Mathematics Seminar

Time:	Friday, 11 March 2016, 2:15 – 3:15 PM
Location:	237 Derrick Hall
Title:	Solution to a Combinatorial Problem arising in Group Theory
Speaker:	Dr. Eugene Curtin, Department of Mathematics

Abstract:

In a 2014 paper Thomas Keller conjectured that given any $n \times \infty$ matrix of n element sets $(S_{i,j})$, it is possible to construct an $n \times \infty$ matrix $(x_{i,j})$ satisfying the following conditions: (i) For all i and j, $x_{i,j} \in S_{i,j}$. (ii) The first $n-2$ elements in each row are distinct and never repeated later in the row. (iii) For all t the n sets $\{x_{i,1}, x_{i,2}, \ldots, x_{i,t}\}$ are distinct.

He proved the $n = 4$ case in his paper, and we will outline a proof for the general case. We will also show the following:

Let X be a subset of the Boolean lattice on $[n]$ satisfying the following conditions: (i) $\{i\} \in X$ for all $i \in [n]$. (ii) For all $A \in X$ with $|A| \leq n-2$ there exist elements $i \neq j$ in $[n] - A$ such that $A \cup \{i\} \in X$ and $A \cup \{j\} \in X$. Then X contains n disjoint chains of length $n-1$.

We conjecture that if $\{X_i\}_{i=1}^n$ is a collection of n subsets of the Boolean lattice on $[n]$ each satisfying (i) and (ii) above then there exist n disjoint chains C_i of length $n-1$ with $C_i \subset X_i$.

This Boolean lattice conjecture implies a stronger version of the infinite matrix result. There is a combinatorial-game version of the conjecture which is stronger still.

No specialized background is needed to follow the arguments. The techniques are elementary, with the Max-Flow Min-Cut Theorem and the Konig Infinity Lemma making guest appearances.

This is joint work with Suho Oh.