TEXAS STATE UNIVERSITY

The rising STAR *of Texas*

Discrete Mathematics Seminar

*** To avoid a schedule conflict with Dr. Hailey's State of the College talk, this talk has been changed to 1:00 - 1:50PM. ***

Time: Friday, 30 March 2018, 1:00 – 1:50 PM
Location: 237 Derrick Hall
Title: Matrix-Tree Generalizations via Modified Grassmann Variables
Speaker: Dr. Eugene Curtin, Department of Mathematics, Texas State University

Abstract:

In a 2008 paper Abdessalem showed how to apply Grassmann-Bezerin calculus to proving a general all-minors matrix tree theorem initially due to Moon. We consider some modifications of the technique. Given any square matrix with entries in any commutative ring, we can associate two weighted directed graphs to the matrix. For one graph, the matrix is a generalized Laplacian matrix, for the other the matrix is a generalized signless Laplacian. We introduce modified Grassmann variables ψ_i and $\bar{\psi}_i$ which for all i and j satisfy $\psi_i^2 = 0$, $\bar{\psi}_i^2 = 0$, and $\psi_i\bar{\psi}_j = \bar{\psi}_j\psi_i$. We consider two cases for the variables, commuting or anti-commuting, where for all i and j the commuting variables satisfy $\psi_i\psi_j = \psi_j\psi_i$ and $\bar{\psi}_i\bar{\psi}_j = \bar{\psi}_j\bar{\psi}_i$. The anti-commuting variables satisfy $\psi_i\psi_j = -\psi_j\psi_i$ and $\bar{\psi}_i\bar{\psi}_j = -\bar{\psi}_j\bar{\psi}_i$. We obtain a generating function in terms of these variables where each determinant (anti-commuting case) or permanent (commuting case) of each square sub-matrix appears as the coefficient of an associated monomial, and show that this coefficient can be computed as a weighted count of subgraphs of either of the two graphs.

This is joint work with former Math Camp students Junu Lee, Andrew Lu and Sophia Sun.