Lower Bounds on the Distance Domination Number of a Graph

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Abstract

For an integer $k \geq 1$, a (distance) k-dominating set of a connected graph G is a set S of vertices of G such that every vertex of $V(G) \setminus S$ is at distance at most k from some vertex of S. The k-domination number, $\gamma_k(G)$, of G is the minimum cardinality of a k-dominating set of G. In this talk, we establish lower bounds on the k-domination number of a graph in terms of its diameter, radius and girth. We prove that for connected graphs G and H, $\gamma_k(G \times H) \geq \gamma_k(G) + \gamma_k(H) - 1$, where $G \times H$ denotes the direct product of G and H.

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