



The rising STAR of Texas

Discrete Mathematics Seminar

Time: Friday, 27 October 2017, 2:15 – 3:15 PM
Location: 237 Derrick Hall
Title: Natural Generalizations of Graphs Part II: Commas, Topoi, & Homomorphisms
Speaker: Dr. William Grilliette, Department of Mathematics, Texas State University

Abstract:

Continuing from Part I, this talk focuses on the abstract construction, properties, and homomorphisms of graph-like objects.

The classical categories \mathfrak{H} of hypergraphs and \mathfrak{M} of (undirected) multigraphs arise naturally as a comma category using the power-set functor \mathcal{P} . However, \mathcal{P} is well-known not to preserve limit processes, and both \mathfrak{H} and \mathfrak{M} fail to be cartesian closed as a result, among other issues.

On the other hand, the category \mathfrak{Q} of quivers arises equivalently as both a comma category and a functor category. Consequently, \mathfrak{Q} can be represented as a topos of presheaves, inheriting a significant amount of structure immediately.

Thus, we suggest another model of hypergraphs, and multigraphs by extension, which is based on incidence rather than adjacency. The category \mathfrak{R} of these “incidence hypergraphs” will be a presheaf topos like \mathfrak{Q} . Indeed, \mathfrak{Q} and \mathfrak{R} are connected by several functors, which seem to encode matricial information into the graph structure itself.