Discrete Mathematics Seminar

Time:	Friday, 8 October 2010, 12:30–1:30 PM
Location:	238 Derrick Hall
Title:	Attainability of the Chromatic Numbers of Functigraphs
Speaker:	Daniel Wang, Liberal Arts and Science Academy, Austin
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Abstract:

Our research examines the chromatic numbers of functigraphs. Functigraphs are generalizations of permutation graphs, introduced by Chartrand and Harary. A functigraph, $C(G, f)$, is obtained from a graph G and a function $f : V(G) \to V(G')$ where G' is a disjoint copy of G. The chromatic number of a graph, $\chi(G)$, is the least number of colors needed to assign a color to each vertex of G such that no two adjacent vertices are given the same color. Recently, *Chen et al.* determined the bounds on $\chi(C(G, f))$ in terms $\chi(G)$. Namely, they proved that

$$\chi(G) \leq \chi(C(G, f)) \leq \left\lceil \frac{3}{2}\chi(G) \right\rceil .$$

We extend upon this by proving that for any two positive integers a and b with $a \leq b \leq \lceil \frac{3}{2}a \rceil$, there exists a graph G and function f on $V(G)$ such that $\chi(G) = a$ and $\chi(C(G, f)) = b$. We then examine the lower and upper bounds of the chromatic number of functigraphs for any graph. For any function f on $V(G)$, we demonstrate sufficient conditions for $\chi(C(G, f)) = \chi(G)$. In fact, the number of such functions is at least $(k - 1)k^{k-1}$, where $k = \chi(G)$.

However, the upper for the chromatic number of functigraph is not always achievable. We improve the upper bounds on $\chi(C(G, f))$ for wheels from $\lceil \frac{3}{2}\chi(G) \rceil$ to $\lceil \frac{3}{2}\chi(G) \rceil - 1$.

We also characterize functions to determine the values of $\chi(C(G, f))$ for all bipartite graphs and odd wheels.