Discrete Mathematics Seminar

Time: Friday, 18 February 2011, 12:30–1:30 PM
Location: 238 Derrick Hall
Title: Monochromatic Square Garden: coloring the integer grid
Speaker: Dr. Jacob Manske, Mathematics Department

Abstract:

For $n \in \mathbb{N}$, let $[n]$ denote the integer set $\{0, 1, \ldots, n-1\}$. For any subset $V \subset \mathbb{Z}^2$, let $Hom(V) = \{cV + \mathbf{b} : c \in \mathbb{N}, \mathbf{b} \in \mathbb{Z}^2\}$. For $k \in \mathbb{N}$, let $R_k(V)$ denote the least integer N_0 such that for any $N \geq N_0$ and for any k-coloring of $[N]^2$, there is a monochromatic subset $U \in Hom(V)$.

The argument of Gallai ensures that $R_k(V)$ is finite whenever V is. We investigate bounds on $R_k(V)$ when V is a three or four-point configuration in general position. In particular, we prove that $R_2(S) \leq VW(8)$, where VW is the classical van der Waerden number for arithmetic progressions and S is a square $S = \{(0,0), (0,1), (1,0), (1,1)\}$.

We will also visit new results including a computer-proven bound which is far, far smaller than the analytic bound achieved in the paper.