Discrete Mathematics Seminar

Time: Friday, 22 March 2013, 1:00 – 2:00 PM
Location: 238 Derrick Hall
Title: Demolishing the previous bounds: an attack on finding representations of Ramsey Algebras
Speaker: Dr. Jacob Manske, Mathematics Department

Abstract:

For a positive integer N, a subset $X \subseteq Z_N$ is called a *cyclic basis* for Z_N if $(X + X) \cup X = \mathbb{Z}_N$. A cyclic basis X is called *sum-free* if $(X + X) \cap X = \varnothing$.

For a positive integer m, a partition of $Z_N \setminus \{0\}$ into sets $X_0, X_1, \ldots, X_{m-1}$ is called a *sum-free cyclic multi-basis* if X_i is a sum-free cyclic basis for $i = 0, 1, \ldots, m-1$.

Related to finding cyclic multi-bases is the *Ramsey Algebra Problem.* Given an integer m, we must find an integer N and partition $X_0, X_1, \ldots, X_{m-1}$ of $\mathbb{Z}_N \setminus \{0\}$ so that

- X_i is closed under additive inverse;
- $X_0, X_1, \ldots, X_{m-1}$ forms a sum-free cyclic multi-basis for Z_N; and
- for $i \neq j$, $X_i + X_j = \mathbb{Z}_N \setminus \{0\}$.

Before the break, Jeremy Alm spoke at the seminar concerning the existence of such partitions. He mentioned constructions for $m = 2$, $m = 3$, and $m = 4$, and that Comer had a construction for $m = 5$, while Maddux had found constructions for $m = 6$ and $m = 7$ (which remain unpublished). All other cases for m remained open, until last weekend.

By noticing a curious pattern, we were able to find constructions for all m up through $m = 175$, except for possibly $m = 8$ and $m = 13$. For $m = 8$, there is no construction which follows this curious pattern, although there may be a construction of a different form.

We will provide an explanation of this pattern, together with some lemmas that make it easier to find constructions. Upon request, the data for the constructions can be provided.