Discrete Mathematics Seminar

Time: Friday, 9 November 2012, 2:00–3:00 PM
Location: 238 Derrick Hall
Title: The chromatic number of the plane and the vector graph
Speaker: Dr. Jacob Manske, Mathematics Department

Abstract:

For $n \in \mathbb{Z}^+$, let $[n] = \{1, 2, \ldots, n\}$. Let $\mathcal{E}_n$ denote the graph whose vertex set is $\mathbb{R}^n$, where two vertices are adjacent if and only if they are distance 1 apart using the standard Euclidean metric. For $k \in \mathbb{Z}^+$, a k-coloring of a graph G is a function $f : V(G) \to [k]$; a k-coloring is *proper* if whenever x and y are adjacent in G, $f(x) \neq f(y)$. The *chromatic number* of a graph G (which we shall denote by $\chi(G)$) is the smallest positive integer c such that there exists a proper c-coloring of G.

The best known bounds for $\chi(\mathcal{E}_2)$ are

$$4 \leq \chi(\mathcal{E}_2) \leq 7,$$

although determining $\chi(\mathcal{E}_2)$ has proven to be an extremely stubborn problem.

In this talk, we provide a survey and history of previous results, as well as current state of the research on related problems. We also introduce the concept of the *vector graph* and a new approach for finding colorings of the plane.