# Upper bounds on the k-forcing number of a graph

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**Abstract**

Given a simple undirected graph $G$ and a positive integer $k$, the $k$-forcing number of $G$, denoted $F_k(G)$, is the minimum number of vertices that need to be initially colored so that all vertices eventually become colored during the discrete dynamical process described by the following rule. Starting from an initial set of colored vertices and stopping when all vertices are colored: if a colored vertex has at most $k$ non-colored neighbors, then each of its non-colored neighbors becomes colored. When $k = 1$, this is equivalent to the zero forcing number, usually denoted with $Z(G)$, a recently introduced invariant that gives an upper bound on the maximum nullity of a graph. In this paper, we give several upper bounds on the $k$-forcing number. Notable among these, we show that if $G$ is a graph with order $n \geq 2$ and maximum degree $\Delta \geq k$, then $F_k(G) \leq \frac{(\Delta-k+1)n}{\Delta-k+1+\min\{\delta,k\}}$. This simplifies to, for the zero forcing number case of $k = 1$, $Z(G) = F_1(G) \leq \frac{\Delta n}{\Delta+1}$. Moreover, when $\Delta \geq 2$ and the graph is $k$-connected, we prove that $F_k(G) \leq \frac{(\Delta-2)n+2}{\Delta+k-2}$, which is an improvement when $k \leq 2$, and specializes to, for the zero forcing number case, $Z(G) = F_1(G) \leq \frac{(\Delta-2)n+2}{\Delta-1}$. These results resolve a problem posed by Meyer about regular bipartite circulant graphs. Finally, we present a relationship between the $k$-forcing number and the connected $k$-domination number. As a corollary, we find that the sum of the zero forcing number and connected domination number is at most the order for connected graphs.

## Short Biographical Sketch

Ryan Pepper received his B.S. in Mathematics (2000), M.S. in Applied Mathematics (2002), and Ph.D. in Mathematics (2004) from the University of Houston. His dissertation was primarily about upper and lower bounds on the independence number for chemical graphs. He spent one year as a post-doctoral research associate at Texas A&M University at Galveston working with Doug Klein in chemical graph theory, before being hired by University of Houston – Downtown, where he is currently an Associate Professor of Mathematics. Recently, his research has been in independence and domination in graphs, degree sequence invariants, and zero-forcing number.

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