Discrete Mathematics Seminar

Time:	Tuesday, 22 January 2013, 2:00 – 3:00 PM
Location:	122 Derrick Hall
Title:	Quadratic and Cubic Forms
Speaker:	Dr. Daniel Shapiro, Department of Mathematics, Ohio State University

Abstract:

Polynomial $Q(X) = 3x_1^2 + 6x_1x_2 - 5x_2^2$ is homogeneous of degree 2: a quadratic form. Then $B(X,Y) = 3x_1y_1 + 3(x_1y_2 + x_2y_1) - 5x_2y_2$ is the associated symmetric bilinear form. This provides a geometric context: the bilinear map $B : V \times V \to \mathbf{R}$ on vector space $V = \mathbf{R}^2$ is viewed as a kind of dot product, with the original form $Q(v) = B(v,v)$ as an analogue of the squared-length of the vector $v = (x_1, x_2)$. This geometric approach to quadratic forms is useful, leading to ideas of orthogonal complements, diagonalization (existence of orthogonal bases), isometries and orthogonal groups, symmetric maps, etc.

How much of this classic theory can be imitated for cubic forms? A cubic form $\varphi(X)$ in n variables does arise from a unique symmetric trilinear form θ. Then $\theta : V \times V \times V \to \mathbf{R}$ is a dot product analogue, with $\varphi(v) = \theta(v,v,v)$. The "center" $Cent(\varphi)$ is the set of maps $f : V \to V$ that are symmetric: $\theta(f(u), v, w) = \theta(u, f(v), w)$ for every u, v, w. This center is a commutative algebra that seems important. We look for examples of cubic forms with non-trivial centers.