Discrete Mathematics Seminar

Time: Friday, 27 April 2012, 12:30-1:30 PM
Room: 238 Derrick Hall
Title: How an Instructor Learn from Students: Three Research Projects from Math 5350 / 7331 (Combinatorics)
Speaker: Dr. Jian Shen, Mathematics Department

Abstract:

This semester I teach Math 5350/7331 (Combinatorics). I assigned some open problems as research projects to students who are eager to explore mathematics by themselves. At the beginning I didn't expect students could really discover something new. But to my surprise I learned a lot from those diligent Math 5350/7331 students. In this talk, I will present three such examples.

1. Recursively define $g^{(n)}(1) = \lceil 3\ g^{(n-1)}(1) / 2 \rceil$ with the initial condition $g^{(0)}(1) =1$. This function was used in an elegant algorithm to determine the winner of the Josephus Problem J_3 (n). (Start with n people around a circle, and eliminate every third remaining person until only one -- the winner -- survives.) The textbook (Concrete Mathematics by Graham et al, Page 81) claimed that "… (the function) probably don't have a nice closed form." Two students (Suresh Srinivasan, Raymond Holland) independently worked on the functional behavior on $g^{(n)}(1)$ hoping to find a closed form for it. In particular, Srinivasan's computational data seemed to suggest that the limit of $g^{(n)}(1) * (2/3)^n \rightarrow 1.622270\ldots$ I have a proof for the statement now.

2. Define $\varepsilon_2(n!)$ (and $\varepsilon_3(n!)$ respectively) to be the largest power of 2 (and 3 respectively) in the unique factorization of n!. For example, $\varepsilon_2(6!) = 4$ and $\varepsilon_3(6!) = 2$ since $6! = 2^4 * 3^4 *5$. Indeed n= 6 is the first positive integer satisfying $\varepsilon_2(n!) = 2 * \varepsilon_3(n!)$. The textbook (Page 114) claimed that "… but nobody has yet proved that such coincidences $\varepsilon_2(n!) = 2 * \varepsilon_3(n!)$ happen infinitely often." Three students (Hassan Rabeti, Webre, Brittany, Suresh Srinivasan) independently worked on finding more such coincidences. In particular, Rabeti wrote a C program that was so powerful that it discovered over 62 million (or 6.2%) such coincidences among the first 1 billion positive integers. Recall that there are nearly 51 million (or 5.1%) prime numbers from 1 to 1 billion. So I conjecture that the number of such coincidences from 1 to n is Θ (n/ ln n), the same order as the number of the primes.

3. Generalized Stern-Brocot tree. Too tired to write a summary on this at 2:45AM now. Please just come to the talk.