Discrete Mathematics Seminar

Time: Friday, 10 February 2012, 12:30–1:30 PM
Location: 238 Derrick Hall
Title: Primitive Normal Matrices and Covering Numbers of Finite Groups (Part I)
Speaker: Dr. Jian Shen, Mathematics Department

Time: Friday, 17 February 2012, 12:30–1:30 PM
Location: 238 Derrick Hall
Title: Primitive Normal Matrices and Covering Numbers of Finite Groups (Part II)
Speaker: Dr. Tim Bonner, Mathematics Department

Abstract:

In this two-talk series, we will cover results from the paper by Chillag, Holzman, and Yona [Primitive normal matrices and covering numbers of finite groups. Linear Algebra and its Applications, 403(2005): 165-177.] We hope to bring collaborations between graph theorists and group theorists in the Department.

In the first talk, Dr. Shen will cover the graph theory part on primitive normal matrices. A primitive matrix is a square matrix M with nonnegative real entries such that the entries of M^r are all positive for some positive integer r. The smallest such r is called the exponent of M, denoted $\exp(M)$. [Here is an equivalent definition for digraphs. A digraph G is primitive if, for some positive integer r, there is a $u \to v$ walk of length r for every pair u, v of vertices of G. The minimum such r is called the exponent of G, denoted $\exp(G)$. So a matrix M is primitive iff the adjacency digraph of M is primitive.] The following two results will be presented:

(1) Let m be the degree of the minimal polynomial of a primitive digraph M, and let D be the diameter of the adjacency digraph of M. Then $D \leq m - 1$.
[Comments: This was the first step towards the bound $\exp(M) \leq D^2 + 1$ which was conjectured by Hartwig in 1984 and proved in 1993 by Dr. Shen (a Master's student at that time). The $(D^2 + 1)$-bound is still the best even today.]
(2) Let M be an $n \times n$ primitive matrix of normal type; that is, $MM^T = M^T M$, where M^T denotes the transpose matrix of M. Then $\exp(M) \leq (\lceil n/2 \rceil + 1)\,(m - 1)$.

In the second talk, Dr. Bonner will cover the group theory part on covering numbers of finite groups.