

Discrete Mathematics Seminar

Time: Friday, 14 October 2011, 1:00–2:00 PM
Location: 329 Derrick Hall
Title: Continued Fractions and Nathanson Heights
Speaker: Dr. Jian Shen, Mathematics Department

Abstract:

Let p be a prime, and let \mathbb{Z}_p denote the field of integers modulo p . The *Nathanson height* of a point $\mathbf{r} = \langle r_1, \dots, r_d \rangle \in \mathbb{Z}_p^d$ is defined as $h_p(\mathbf{r}) = \min\{\sum_{i=1}^d (kr_i \bmod p) : k = 1, \dots, p-1\}$. In 2007 Sullivan and Nathanson applied the concept of Nathanson height in their work on a special case (Cayley digraphs $\text{Cay}(\mathbb{Z}_p, \{r_1, \dots, r_d\})$) of a graph theory conjecture raised by Chudnovsky, Seymour, and Sullivan.

An explicit formula for the Nathanson height is not known for general d . For $d = 2$ O'Bryant recently proved that $h_p(\langle 1, r \rangle) = \min\{q_{2k}(r+1) - p_{2k}p : 0 \leq k < n/2\}$, where p_i/q_i is the i^{th} convergent of the continued fraction expansion of r/p . We study the properties of continued fractions and further simplify the above expression for $h_p(\langle 1, r \rangle)$.

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