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Discrete Mathematics Seminar

Time: Friday, September 13, 2019, 2:15-3:15 PM
Room: 330 Derrick Hall
Title: On the Sensitivity Conjecture
Speaker: Dr. Jian Shen, Department of Mathematics, Texas State University

Abstract:

This past summer, Hao Huang (Assistant Professor in the Department of Mathematics and Computer Science at Emory University) gave a remarkably short proof of an 3-decade long-standing problem in theoretical computer science known as the sensitivity conjecture. See below for a media report: https://www.quantamagazine.org/mathematician-solves-computer-science-conjecture-in-two-pages-20190725/ A comment made by Scott Aaronson (Computer Science Professor at UT-Austin) said "… the list of people who tried to solve it and failed is like a who's who of discrete math and theoretical computer science."

In this talk, I will go over details of Huang's proof. His proof involved an elegant construction of a sequence of matrices which mimic the adjacency matrices of hypercubes. Then he estimated the eigenvalues of these matrices using techniques such as the Cauchy interlace theorem. His main theorem is as follows: "For any subset S of more than half the vertices in an n-dimensional hypercube, there is always some vertex that is adjacent to at least $\sqrt{n}$ of the other vertices in S." Then the sensitivity conjecture

followed immediately from this result (with the help of some earlier results proved by other computer scientists).

Sensitivity Conjecture. The sensitivity of a Boolean function f(x) of n Boolean variables is the maximum over all inputs x of the number of positions i such that flipping the i-th bit of x changes the value of f(x). Permitting to flip disjoint blocks of bits leads to the notion of block sensitivity, known to be polynomially related to a number of other complexity measures of f, including the decision-tree complexity, the polynomial degree, and the certificate complexity. A long-standing open question is whether sensitivity also belongs to this equivalence class. A positive answer to this question is known as the Sensitivity Conjecture.