Discrete Mathematics Seminar

Time: Friday, 14 October 2016, 2:15 – 3:15 PM
Location: 237 Derrick Hall
Title: THOMPSON'S PROBLEM AND THOMPSON'S CONJECTURE
Speaker: Dr. Wujie Shi, School of Mathematics and Statistics, Chongqing University of Arts and Sciences, P.R. China

Abstract: Let G be a finite group and $\pi_e(G) = \{o(g) \mid g \in G\}$, that is, the set of all element orders for G. In 1987, we put forward the following conjecture:

Conjecture. All finite simple groups G can be determined uniformly using their orders $|G|$ and their element orders $\pi_e(G)$.

Now this conjecture was proved and became a theorem. A problem related to the above conjecture is the following open problem put forward by J.G. Thompson in 1987: For each finite group G and each integer $d \geq 1$, let $G(d) = \{x \in G; x^d = 1\}$.

Definition. G_1 and G_2 are of the same order type if and only if $|G_1(d)| = |G_2(d)|$, $d = 1, 2, \cdots$.

Thompson's Problem. Suppose G_1 and G_2 are of the same order type. Suppose also that G_1 is solvable. Is it true that G_2 is also necessarily solvable?

In Thompson's letter he pointed out that "The problem arose initially in the study of algebraic number fields, and is of considerable interest".

Another Thompson's conjecture, which was also aim at characterizing all finite simple groups by a quantity set, was posed in 1988, which appeared in another communication letter to the author:

If G is a finite group, set $N(G) = \{n \in Z^+, G$ has a conjugacy class C with $C = n\}$.

Thompson's Conjecture. If G and M are finite groups and $N(G) = N(M)$, and if in addition, M is a non-Abelian simple group while the center of G is 1, then G and M are isomorphic.

In this talk we will discuss the above Thompson's problem and Thompson's conjecture.