Discrete Mathematics Seminar

Time: Friday, 8 April 2011, 2:15–3:15 PM
Location: 238 Derrick Hall
Title: The relative distances of points in the boundary of plane convex bodies
Speaker: Dr. Zhanjun Su, Department of Mathematics, Hebei Normal University, China

Abstract

Given $k \geq 2$, finding k points on the sphere or in the ball of a Euclidean n-space E^n such that their pairwise distances are as large as possible is a long-standing problem in geometry. A generalization of this problem was presented by Doyle, Lagarias and Randall, and by Lassak. Doyle, Lagarias and Randall considered the points in the boundary of the centrally symmetric bodies. Lassak gave a more general approach. Here C is an arbitrary convex body and the problem is to find configurations of points in the boundary of C, whose pairwise distances are large in the sense of the following notion of C-distance of points. Let C be a plane convex body. The relative distance of points $a, b \in C$ is the ratio of the Euclidean distance of a and b to the half of the Euclidean distance of $a_1, b_1 \in C$ such that a_1b_1 is a longest chord of C parallel to the line-segment ab. Denote by $\mu_k(C)$ the greatest possible number d such that the boundary of C contains k points at pairwise C-distance at least d, and denote by $\mathcal{C}$ the family of plane convex bodies. Let $\mu_k(\mathcal{C}) = \sup\{\mu_k(C) \mid C \in \mathcal{C}\}$. It was conjectured that $\mu_{10}(\mathcal{C}) = \mu_{11}(\mathcal{C}) = \frac{2}{3}$. In this paper we show that $\mu_{10}(\mathcal{C}) > \frac{2}{3}$ and prove that $\mu_{11}(\mathcal{C}) = \frac{2}{3}$ holds.