

## Discrete Mathematics Seminar

Time: Friday, 1 November 2013, 1:00 – 2:00 PM  
Location: 238 Derrick Hall  
Title: The Borsuk-Ulam Theorem and a Variation on the Game of Hex  
Speaker: Leslie Tu, Westwood High School, Austin  
Selcen Yuksel, Harmony Science Academy, Houston  
Mentor: Dr. Eugene Curtin, Mathematics Department

### **Abstract:**

A special tradition in mathematics is using games and puzzles to approach formidable problems. Our research begins by examining the Hex Theorem, which leads directly to a proof of the famous Brouwer Fixed-Point Theorem. We alter the rules of Hex to create a new game called “Tri,” played on a triangulation of the sphere. The Tri board is a centrally symmetric triangulation of the sphere; two players, red and blue, alternate coloring a vertex and its antipode with the ultimate goal of creating a monochrome path connecting antipodal points. This game is structurally similar to Hex but can encompass far more general and disorderly cases.

We prove that Tri must have exactly one winner by induction on the number of vertices. We also show how Hex is a special case of Tri and thus provide a new proof of the Hex Theorem. These results are the first steps in our new, elementary proof of the Borsuk-Ulam Theorem in topology.