## **Discrete Mathematics Seminar**

Time:	Friday, 31 January 2014, 1:00 – 2:00 PM
Location:	238 Derrick Hall
Title:	Second Order $(\Phi, \rho, \eta, \theta)$ -Invexities and Parameter-Free $\epsilon$ -Efficiency Conditions
	for Multiobjective Discrete Minmax Fractional Programming Problems
Speaker:	Dr. Ram Verma, Mathematics Department

## Abstract:

First a general framework for a class of second order  $(\Phi, \rho, \eta, \theta)$ -invexities is introduced, and then some parameter-free sufficient efficiency conditions leading to  $\epsilon$ -efficient solutions to multiobjective discrete minmax fractional programming problems of the form (P)are established. The obtained results generalize and unify a wider range of investigations in the literature on applications to multiobjective fractional programming, while these findings can be utilized as a resource in order to measure the efficiency or productivity of a system. We consider based on the generalized  $(\Phi, \rho, \eta, \theta)$ -invexities of functions, the following multiobjective fractional programming problem:

 $(\mathbf{P})$ 

$$Minimize\left(\frac{f_1(x)}{g_1(x)}, \frac{f_2(x)}{g_2(x)}, \cdots, \frac{f_p(x)}{g_p(x)}\right)$$

subject to  $x \in Q = \{x \in X : H_j(x) \leq 0, j \in \{1, 2, \dots, m\}\}$ , where X is an open convex subset of  $\Re^n$  (n-dimensional Euclidean space),  $f_i$  and  $g_i$  for  $i \in \{1, \dots, p\}$  and  $H_j$  for  $j \in \{1, \dots, m\}$  are real-valued functions defined on X such that  $f_i(x) \geq 0, g_i(x) > 0$  for  $i \in \{1, \dots, p\}$  and for all  $x \in Q$ . Here Q denotes the feasible set of (P).

The obtained results can also be applied to discrete minmax multiobjective fractional integral programming (optimal control) problems of the form  $(P^*)$ .

Consider the fractional programming for variational problem of the following form:

$$(\mathbf{P}^*) \qquad \qquad \min_{x} \left\{ \sum_{1 \le i \le p} \max \frac{\int_{a}^{b} f^{i}(t, x, \dot{x}) dt}{\int_{a}^{b} g^{i}(t, x, \dot{x}) dt} \right\}$$
  
subject to  $x \in PS(T, \mathbb{R}^{n}), x(a) = \alpha, x(b) = \beta,$ 
$$\int_{a}^{b} h^{j}(t, x, \dot{x}) dt \le 0, j \in \mathbf{m} \equiv \{1, 2, \dots, m\},$$

where functions  $f^i, g^i, i \in \mathbf{p}$  and  $h^j, j \in \mathbf{m}$  are continuous in t and have continuous partial

derivatives with respect to x and  $\dot{x}$ ; T = [a, b] denotes the time space, and  $PS(T, R^n)$  stands for the state space of all piecewise smooth functions  $x : T \to R^n$  with norm defined by  $||x|| = ||x||_{\infty} + ||Dx||_{\infty}$  and D is the differential operator on  $PS(T, R^n)$  defined by

$$y = Dx$$
 if and only if  $x(t) = x(a) + \int_{a}^{b} y(s)ds$ .