# Discrete Mathematics Seminar

Time: Friday, 13 March 2015, 2:00 – 3:00 PM
Location: 237 Derrick Hall
Title: On p-part of character degree of p-solvable groups
Speaker: Dr. Yong Yang, Mathematics Department

**Abstract:**

Let $G$ be a finite group and $P$ be a Sylow $p$-subgroup of $G$, it is reasonable to expect that the degrees of irreducible characters of $G$ somehow restrict those of $P$. The Ito-Michler theorem proves that each ordinary irreducible character degree is coprime to $p$ if and only if $G$ has a normal abelian Sylow $p$-subgroup. Of course, this implies that $|G : \mathbf{F}(G)|_p = 1$.

Let $\mathrm{Irr}(G)$ be the set of irreducible complex characters of $G$, and $e_p(G)$ be the largest integer such that $p^{e_p(G)}$ divides $\chi(1)$ for some $\chi \in \mathrm{Irr}(G)$. Isaacs [1] showed that if $G$ is solvable, then the derived length of a Sylow $p$-subgroup of $G$ is bounded above by $2e_p(G) + 1$.

Let $b(P)$ denote the largest degree of an irreducible character of $P$. [2, Conjecture 4] suggested that $\log b(P)$ is bounded as a function of $e_p(G)$. Moretó and Wolf [3] have proven this for $G$ solvable and even something a bit stronger, namely the logarithm to the base of $p$ of the $p$-part of $|G : \mathbf{F}(G)|$ is bounded in terms of $e_p(G)$. In fact, they showed that $|G : \mathbf{F}(G)|_p \leq p^{19e_p(G)}$ for any solvable groups [3, Corollary B (i)], and $|G : \mathbf{F}(G)|_p \leq p^{2e_p(G)}$ for odd order groups [3, Corollary B (iii)].

In this talk, we show that for $p$-solvable groups, $|G : \mathbf{F}(G)|_p \leq p^{ke_p(G)}$ for some constant $k$. This implies [2, Conjecture 4] for $p$-solvable groups.

## References

[1] I.M. Isaacs, 'The $p$-parts of character degrees in $p$-solvable groups', Pacific J. Math. 36 (1971), 677C691.

[2] A. Moretó, 'Characters of $p$-groups and Sylow $p$-subgroups', Groups St. Andrews 2001 in Oxford, Cambridge University Press, Cambridge, 412-421.

[3] A. Moretó and T.R. Wolf, 'Orbit sizes, character degrees and Sylow subgroups', Advances in Mathematics 184 (2004), 18-36.