

TRANSFER OF CLASSICAL THEOREMS ON GROUPS AND GEOMETRIES TO ASSOCIATION SCHEMES

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Let X be a set, and let S be a partition of the cartesian product $X \times X$. Assume that $1_X \in S$ and that, for each element s in S , $s^* := \{(y, z) \mid (z, y) \in s\} \in S$. For any two elements x in X and s in S , define $xs := \{y \in X \mid (x, y) \in s\}$. The set S is called a scheme if, for any three elements p, q , and r in S , there exists a cardinal number a_{pqr} such that, for any two elements y and z in X , $(y, z) \in r$ implies that $|yp \cap zq^*| = a_{pqr}$.

The concept of an association scheme generalizes not only the notion of a group but also the one of a building (in the sense of Jacques Tits) along with quite a few other mathematical concepts (distance regular graphs, block designs with $\lambda = 1$, Moore geometries, etc.). Various basic results on finite groups have been generalized to association schemes, among them Lagrange's Theorem, the Homomorphism Theorem and the Isomorphism Theorems, the Jordan-Hölder Theorem, Sylow's Theorems, and recently part of the Schur-Zassenhaus Theorem. In my talk, I will review some of these theorems.

The buildings come into the game if one generalizes the notion of an involution.

For any two elements p and q of a scheme S , we define pq to be the set of all elements s in S such that $1 \leq a_{pqs}$. A non-empty subset T of a scheme S is called closed if $p^*q \subseteq T$ for any two elements p and q in T .

For each subset R of a scheme S , we define $\langle R \rangle$ to be the intersection of all closed subsets of S which contain R as a subset. An element s in a scheme S is called an involution if $|\langle \{s\} \rangle| = 2$.

Let S be an association scheme, let L be a set of involutions of S , and assume that $\langle L \rangle = S$. The scheme S is called a Coxeter scheme with respect to L if it is constrained with respect to L and if L satisfies the exchange condition (a word by word translation of the group theoretic exchange condition to scheme theory).

It has been shown in [2; Sections 6.3, 6.4, 6.5] that Coxeter schemes can be identified with buildings (in the sense of Jacques Tits). In [1; Theorem 12.3.4] it was shown that finite Coxeter schemes are quotients of thin schemes if they do not contain nontrivial thin elements and if the underlying set of involutions has at least three elements. This result is equivalent to Tits' theorem on spherical buildings.

References

- [1] Zieschang, P.-H.: *Theory of Association Schemes*. Springer Monographs in Mathematics, Berlin Heidelberg New York (2005)
- [2] Zieschang, P.-H.: Hypergroups. Preprint (Max-Planck-Institut für Mathematik Preprint Series 2010 (97))

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