Discrete Mathematics Seminar

Time: Friday, 9 October 2009, 1:00–2:00 PM
Location: 238 Derrick Hall
Title: Products of Commutators and the Order of a Finite Group
Speaker: Dr. Timothy Bonner, Mathematics Department

Abstract:

Let G be a finite group. A commutator of G is an element of the form $a^{-1}b^{-1}ab$ where $a, b \in G$. The commutator subgroup of G, denoted G', is the subgroup generated by the set of all commutators. It is known that each element of the commutator subgroup need not be a commutator, but only a product of commutators. We define $\lambda(G)$ to be the minimal integer such that each element of G' may be expressed as a product of $\lambda(G)$ commutators. Modifying a technique of P.X. Gallagher, we show,

$$|G'| \geq (\lambda(G)+1)!\,(\lambda(G)-1)!.$$

This improves the earlier bound of Gallagher (1965),

$$|G'| \geq \frac{1}{2}\,(\lambda(G)+1)!\,(\lambda(G)-1)! + 1.$$

Further, in the most recent edition of the Kourovka Notebook of Unsolved Problems in Group Theory, V.G. Bardakov conjectured that for any finite group G,

$$\frac{\lambda(G)}{|G|} \leq \frac{1}{6},$$

with the bound attained only at the symmetric group on three letters, S_3. We verify his conjecture, and using our improvement of Gallagher's result, we show that if $|G| \geq 1000$,

$$\frac{\lambda(G)}{|G|} \leq \frac{1}{250}.$$

Explicit values of $\lambda(G)$ have been determined for all groups of smaller order. We will also give a brief introduction to some of the key ideas that we use from the character theory of finite groups.