Discrete Mathematics Seminar

Time: Friday, 4 December 2009, 1:00-2:00 PM
Room: 238 Derrick Hall
Title: Counting curves through points with multiplicities
Speaker: Dr. Zach Teitler, Mathematics Department, Texas A&M University

Abstract:

Let A be a set of points in the plane with multiplicities. How many curves of a given degree pass through the points of A with the assigned multiplicities?
This is related to a certain interpolation problem: finding a polynomial of a given degree with some given values and derivatives (Taylor coefficients) at the points of A. If there is a unique curve answering the question above, then interpolation problems on A have a unique solution. This question is also important in algebraic geometry, where the number of curves of each degree is called the Hilbert function of A. For any ideal, its Hilbert function is an important and well-studied invariant. Our goal is to study this invariant for sets of points with multiplicities. We give upper and lower bounds for the Hilbert function of A in terms of knowing which sets of points in A are collinear --- i.e., in terms of the matroid of A. Our method still gives bounds if only some of the collinear sets are specified. This leads to surprisingly good bounds and a simple condition for the bounds to coincide, so in many cases we get an exact computation of the Hilbert function of A. Furthermore when the condition is met, we give upper and lower bounds for the graded Betti numbers, along with a condition for these bounds to coincide.

This is joint work with Susan Cooper and Brian Harbourne. The talk will not assume any background. In particular I will explain the interpolation problem described above and the definition of a Hilbert function.