REGULAR ORBITS OF NILPOTENT SUBGROUPS OF SOLVABLE GROUPS

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ABSTRACT. We settle a conjecture by Walter Carlip [2, Conjecture 1.3]. Suppose G is a finite solvable group, V is a faithful $\mathbb{F}G$-module over a field of characteristic p and assume $O_p(G) = 1$. Let H be a nilpotent subgroup of G and assume that H involves no wreath product $Z_r \wr Z_r$ for $r = 2$ or r a Mersenne prime, then H has at least one regular orbit on V.

1. INTRODUCTION

Let G be a finite group and V a finite faithful $\mathbb{F}G$-module, $\mathbb{F}$ a finite field. The existence of regular orbit of G on V has been studied extensively in the literature. Berger proved the following result [1, Theorem 2.2]. Let G be a finite nilpotent group and let V be a finite faithful G-module with characteristic p, assume $(|G|, p) = 1$ and G involves no wreath product $Z_r \wr Z_r$ for all primes r, then G has a regular orbit on V. Hargraves extended Berger's result as the following [4, Theorem 8.2]. Let G be a finite nilpotent group and let V be a faithful $\mathbb{F}G$-module over a field of characteristic p such that $(|G|, p) = 1$. Assume that G involves no wreath product $Z_2 \wr Z_2$, assume also G involves no wreath product $Z_r \wr Z_r$ for r a Mersenne prime when $p = 2$, then G has a regular orbit on V. When $\text{char}(F)$ divides $|G|$, the picture is much less clear. In [3, p4] Espuelas proved the following result. Let G be a finite solvable group and V be a finite faithful G-module with characteristic p, let H be a p-subgroup of G and $O_p(G) = 1$. If p is an odd prime or H involves no $Z_2 \wr Z_2$, then H has a regular orbit on V. Espuelas also asked whether his result can be extended to H nilpotent. Walter Carlip proved the following result [2, Theorem 1.2] as a partial answer to Espuelas' question. Suppose G is a finite solvable group and V is a faithful $\mathbb{F}G$-module over a field of characteristic p, assume $O_p(G) = 1$, $|G||V|$ is odd and H is a nilpotent subgroup of G, then H has a regular orbit on V. In the same paper, Carlip conjectured the following statement [2, Conjecture 1.3]. Suppose G is a finite solvable group and V is a faithful $\mathbb{F}G$-module over a field of characteristic p, assume $O_p(G) = 1$ and H is a nilpotent subgroup of G where H involves no wreath product $Z_r \wr Z_r$ for $r = 2$ or r a Mersenne prime, then H has at least one regular orbit on V. In this paper, we prove a slight generalization of his conjecture. We prove the following result. Suppose G is a finite solvable group, V is a faithful $\mathbb{F}G$-module over a field of characteristic p and assume $O_p(G) = 1$. Let H be a nilpotent subgroup of G and assume that H involves no wreath product $Z_2 \wr Z_2$, also H involves no wreath product $Z_r \wr Z_r$ for r a Mersenne prime when $p = 2$, then H has at least one regular orbit on V. Our theorem includes all of the previously mentioned results as special cases.

References

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