Groups with an Irreducible Character of Large Degree

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If G is a finite group and χ is an irreducible character of G with degree d, then d divides |G| and $|G| \ge d^2$. Thus we can write |G| = d(d + e) where e is a non-negative integer. It was proved by Isaacs that for e > 1, |G| is bounded by Be^6 where B is some universal unknown constant. In this talk, I will discuss work that I have done with S. Jensen to improve this bound to $|G| < e^6 - e^4$. I will also discuss a generalization of this problem which I have been working on for my thesis. In this generalization, we work relative to a normal subgroup N and write |G : N| = d(d + e) where d is a ratio of character degrees of G and N and e is a non-negative integer. We can then ask whether or not |G : N| is bounded as a function of e for e > 1.